Basement relief delineation of the Marajó Basin using Total Variation inversion constraint

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Abstract

In this study, we present the inversion of gravity data from the Marajó Basin through the iterative 2D gravity inversion effective method for estimating the basement relief of sedimentary basins developed on an extensional tectonic regime. The stability and uniqueness of the proposed method are achieved by introducing a priori information of the discontinuities of the basement relief. Mathematically, this information is introduced efficiently through the Total Variation (TV) stabilizing functional on the parameters to be estimated. The minimization of this constraint favors the mapping of rift basins basement relief, with abrupt variations between spatially adjacent depths. We assume that the basin has discontinuous topography of the basement and known hyperbolic density contrast. The basement relief estimates were obtained assuming that the sedimentary layers can be approximated by an interpretative model consisting of a set of juxtaposed 2D vertical prisms whose thickness of the prisms locally represent the depth of the basement and are the parameters to be estimated via the inversion method. The results obtained were satisfactory mapping basement high angle faults of two profiles of Marajó basin.

Introduction

The growing search for hydrocarbon reserves leads to the study of geological structures found in sedimentary basins that generate and store these resources. The location of topographical features and basement faults related to stratigraphic or structural traps have become the target of geological and geophysical research as they are structures conducive to the accumulation of oil and gas. The mapping of these structures can be obtained efficiently by applying the inversion method to gravity data, establishing an interpretive model based on a priori geological and geophysical information on the study area, under the condition that the misfit generated between gravity observations and the anomaly produced by the interpretive model is minimized.

However, the gravity inverse problem of estimating the basement relief is in general an ill-posed problem (Hadamard, 1902) because its solution is neither unique nor stable due to the attempt to extract more information than is contained in the data. Mathematically, uniqueness and stability can be achieved simultaneously by introducing a priori geological information about the study area through the Tikhonov regularization method (Tikhonov and Arsenin, 1977). This method is based on translating geological information into regularizing functional that are mathematical expressions which impose restrictions on the parameters to be estimated. The functional must be properly designed so that it reaches a minimum when the solution exhibits the expected real geological characteristic; otherwise, the solution may be stable, but far from the true basement topography.

The environment of sedimentary basins is characterized by presenting well-defined geological contacts between the sedimentary package and the basement due to the density contrast. Initially, constant density values for the sedimentary package were widely used (Bott, 1960; Corbató, 1965; Oldenburg, 1974; Leão et al., 1996; Barbosa et al., 1997). More elaborate interpretive models were introduced when they assumed that the sedimentary package is heterogeneous caused by the burial and compaction action resulting from the continuous application of the lithostatic pressure that produces a reduction in the volume of the sediments and, consequently, variation of the density contrast with the depth. Rao (1986), Litinsky (1989) and Rao et al. (1994) established analytical expressions for the gravitational anomaly produced by sources exhibiting respectively quadratic, hyperbolic and parabolic variations in the contrast of density with depth, which allow for more realistic modeling of the sedimentary layers.

Sedimentary basins wichi wered developed under extension forces, such as the Marajó Basin in northern Brazil, are characterized by presenting basement topographies with the presence of high angle faults. In order to estimate the basement relief of this type of basin, we can direct the solution to present a basement relief with the presence of faults through the Total Variation (TV) stabilizing functional.

Despite the intensification of geological studies in the Marajó Basin, more accurate characterization of the basement relief requires further studies, in view of its importance for the oil and gas industry. In this context, this work aims to contribute more geophysical information on the basement topography by applying the 2D gravity inversion method to two gravity sections of the Marajó Basin whose sediment density contrast is known expressed according to a hyperbolic law.

Marajó Basin Settings

The Marajó Basin is located in the North of Brazil, state of Pará, covering an area of 53000 km^2 distributed in four sub-basins called Mexiana, Limoeiro, Cametá and Mocajuba (Villegas, 1994). Regionally, the NW is limited by the bundle of transient faults from Mexiana; the SE through the Tocantins Arch that separates it from the Grajaú Basin and; to the west by the Gurupá Arch, which separates it from the Amazonas Basin (Figure 1).

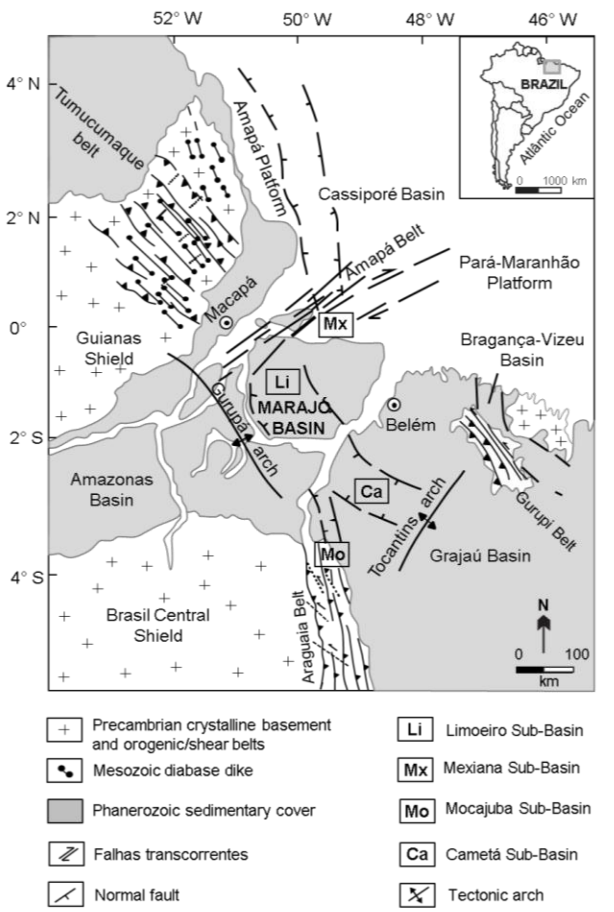

Figure 1 - Simplified tectonic setting of northern Brazil with location of the Marajó Rift and the Precambrian orogenic or shear belts (modified from Costa et al., 2002).

The Marajó Basin developed on the platforms of Pará and Bragantina after the opening of the South Atlantic Ocean and the rupture of Gondwana that led to the separation of the continents of Africa and South America (Zalán and Matsuda, 2007; Júnior et al., 2008; Soares Júnior et al., 2011). The tectonic efforts that initiated the creation of the rift ceased before any exposure of the oceanic crust, therefore, the Marajó Basin is formally classified as an aborted rift, formed by a system of interconnected grabs.

The tectonic scenario of the area around the Marajó Basin (Figure 1) is characterized by the orogenic or shear belts Araguaia and Tumucumaque that were interpreted as a result of pre-Cambrian continental collisions (Hasui et al., 7 1993). The continuity of the Araguaia and Tumucumaque belts, respectively, towards north-northwest and southeast, coincides with the area where the Marajó Basin developed.

The geometric structural framework of the Marajó Basin is defined by asymmetric semi-grids with a center controlled by main normal faults NW-SE, cut by faults occurring in NE-SW and ENE-WSW directions, which limit the two main sub-basins, Limoeiro and Cameta. The northern limit of the Limoeiro Sub-Basin is marked by a beam of transient faults in the NE-SW direction, along which the Mexiana Sub-Basin developed. The Mocajuba Sub-Basin has normal NNW-SSE faults and is separated from the Cametá Sub-Basin by an isolated horst (Costa et al., 2002). According to Lima (1987) the sedimentary layers reach a thickness of up to 11 km.

Method

Forward problem

Let $\mathbf{g}^0$ be an N-dimensional vector of gravity observations referred to a dextral system of Cartesian coordinates (Figure 2a). These observations are produced by a sedimentary basin that presents basement topography with local discontinuities whose sediment density contrast, in relation to the basement, is known and variable with the depth expressed by the hyperbolic law given by (Rao et al., 1994)

$$\Delta\rho(z) = \frac{\Delta\rho_0 \beta^2}{(\beta + z)^2}, \qquad (1)$$

where $\Delta\rho_0$ is the density contrast on the Earth's surface expressed in g/ cm^3, z is the depth and β is a factor that controls the rate of change of the density contrast with the depth expressed in units of length.

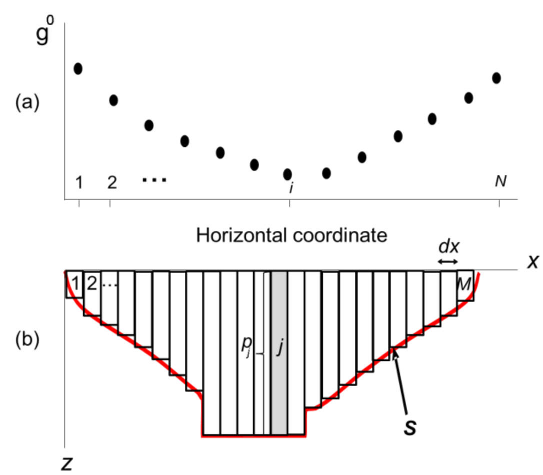

Figure 2 - Gravity anomaly profile (a) produced by a 2D sedimentary basin (b) whose sedimentary package has been discretized into a set of vertical, juxtaposed 2D vertical prisms. The thicknesses $\boldsymbol{p}_j$, j = 1, ..., M of the prisms represent the basement depths (line S).

To estimate the relief of the basement S of this basin, we discretized the sedimentary package in a set of M, 2D, rectangular juxtaposed prisms, with tops positioned on the earth's surface and constant horizontal dimensions (Figure 2b). The prisms' thicknesses, p_j, j = 1, …, M, locally define the depth of the basement, and are the parameters to be estimated from the set of gravity observations.

The thicknesses of the set of M prisms of the interpretive model represent the depths of the basement at discrete points, p_j, related to the N-dimensional vector **g**, whose ith element, g_i, is the theoretical gravity anomaly generated at the ith observation point. The elements g_i and p_j are related by

$$g_i \equiv \mathbf{g}(x_i, \mathbf{p}, \Delta\rho) = \sum_{j=1}^{M} \mathbf{F_P}\,(x_i, \mathbf{x}_j^P, \mathbf{z}_j^P, \Delta\rho), \qquad (2)$$

where $\mathbf{F_P}(x_i, \mathbf{x}_j^P, \mathbf{z}_j^P, \Delta\rho)$ is a nonlinear function, adapted from Rao et al. (1994), to produce the gravity anomaly due to a prism composing the interpretive model of a quadrilateral whose coordinates of the vertices are stored in the vectors $\mathbf{x}_j^P \equiv [x_n^P]$ e $\mathbf{z}_j^P \equiv [z_n^P]$, n = 1, ..., 4. The vector $\Delta\rho$ contains the parameters necessary to define the hyperbolic decay of the density contrast with the depth given in equation 1.

The inverse problem

The geophysical inverse problem of estimating the M-dimensional vector **p** of thickness of the prisms from the observed data is ill-posed problem due to the instability of the solution and the possibility of non-uniqueness. Mathematically, uniqueness can be achieved by incorporating a priori information on the geology of the study area, which simultaneously introduces stability to the solution. As we assumed in the previous section that the basement topography has local discontinuities, we incorporate this information through the Total Variation (TV) function, which simultaneously introduces uniqueness and stability, imposing discontinuities between spatially adjacent parameters. Therefore, we formulate the inverse problem by minimizing the functional

$$\phi^{\mathbf{TV}}(\mathbf{p}) = \frac{1}{M-1}\|\mathbf{Rp}\|_1, \qquad (4)$$

subject to

$$\phi^{\mathbf{g}}(\mathbf{p}) = \frac{1}{N}\left\|\mathbf{g}^0 - \mathbf{g}\right\|^2 = \delta, \qquad (5)$$

Where **R** is a matrix (M - 1) × M of first discrete derivatives whose lines contain only two non-null elements equal to 1 and –1, located in the columns corresponding to the adjacent parameters, δ is the mean square error that contaminates the observations, N is the number of observations, $\mathbf{g}^0$ and **g** are N-dimensional vectors that contain the gravity observations presumably produced by the basement relief of a sedimentary basin and the theoretical gravity anomaly produced by M prisms computed through equation 2, respectively.

The minimization of $\phi^{\mathbf{TV}}(\mathbf{p})$ favors a relief of the basement with the presence of discontinuities; therefore it is suitable for the interpretation of data produced by sedimentary basins developed in an extensional tectonic regime, characteristic of rift basins (Martins et al., 2011; Lima et al., 2011; Santos et al., 2015). We produce the solutions of equations (4) and (5), minimizing the objective function

$$\phi(\mathbf{p}) = \phi^{\mathbf{g}}(\mathbf{p}) + \mu \sum_{i=1}^{M} \sqrt{[\mathbf{Rp}]^2 + \alpha^2}, \qquad (6)$$

where α is a small value (of the order of 10^{-4}) introduced to restore the differentiability of the objective function (Bertete-aguirre; Cherkaev; Oristaglio, 2002) and μ is a positive scalar, called the regularization parameter , which controls the stability of the solution.

The problem of minimizing expression 6 is solved using the Gauss-Newton iterative method using the Marquardt (1963) strategy, as proposed by Silva et al. (2001). From the knowledge of the initial estimate $\hat{\mathbf{p}}^k$ in the kth iteration, the new vector of estimates $\hat{\mathbf{p}}^{k+1}$ is obtained in the k-the first iteration expressed by

$$\hat{\mathbf{p}}^{k+1} = \hat{\mathbf{p}}^k + \Delta\hat{\mathbf{p}}^k, \qquad (7)$$

with the step $\Delta\hat{\mathbf{p}}^k$ expressed as

$$\Delta\hat{\mathbf{p}}^k = \mathbf{D}\left[\mathbf{D}\left(\mathbf{A}_k{}^T\mathbf{A}_k + \lambda\mathbf{R}^T\mathbf{R}\right)\mathbf{D} + \lambda_M\mathbf{I}\right]^{-1}\mathbf{D}\left[\mathbf{A}_k{}^T\Delta\mathbf{g}_k - \lambda\mathbf{R}^T\mathbf{R}\mathbf{p}_k\right], \qquad (8)$$

where the superscript T stands for transposition, λ_M is a positive scalar automatically modified in each iteration, called the Marquardt parameter, **I** is the identity matrix of dimension $M \times M$, $\Delta\mathbf{g}$ is the N-dimensional vector of residues between the observations $\mathbf{g}^0$ and the theoretical data **g** and **D** is a normalizing diagonal matrix of dimension $M \times M$ whose kth element d_{ii}^k is expressed as

$$d_{ii}^k = \left(\frac{1}{c_{ii}^k}\right)^{-\frac{1}{2}} \qquad (9)$$

and c_{ii}^k is the diagonal element of matrix **C**, defined as

$$\mathbf{C} = \mathbf{A}_k{}^T\mathbf{A}_k + \lambda\,\mathbf{R}^T\mathbf{R}, \qquad (10)$$

where **A** is a Jacobian matrix $N \times M$ of the function g_i (equation 2), evaluated in $\hat{\mathbf{p}}^k$ whose element a_{ij}^k is given by

$$a_{ij}^k = \left.\frac{\partial g_i}{\partial p_j}\right|_{p_j = \hat{p}_j^k}. \qquad (11)$$

The iterative process is interrupted when we reach the stopping criterion, calculated from the relative variation of the objective function $\phi(\mathbf{p})$ between successive iterations, expressed by

$$\left|\frac{\phi\left(\hat{\mathbf{p}}^{k+1}\right) - \phi\left(\hat{\mathbf{p}}^k\right)}{\phi\left(\hat{\mathbf{p}}^{k+1}\right)}\right| \le \varepsilon, \qquad (12)$$

where $\phi\left(\hat{\mathbf{p}}^{k+1}\right)$ and $\phi\left(\hat{\mathbf{p}}^k\right)$ are objective functions (equation 6), evaluated in the first and kth iterations, respectively. If the relative variation of the objective function $\phi(\mathbf{p})$ between successive iterations is equal to or less than a limit value ε, defined by the interpreter, the iteration is interrupted.

Results

The data used for the development of this work were obtained on the public data website of BGI (Bureau Gravimétrique International) which provides relative and absolute measurements of the Earth's gravitational field. In this work, we applied the inversion method to two representative profiles A and B located between the sub-basins of Cametá and Mocajuba (Figure 3) in order to delineate the basement relief.

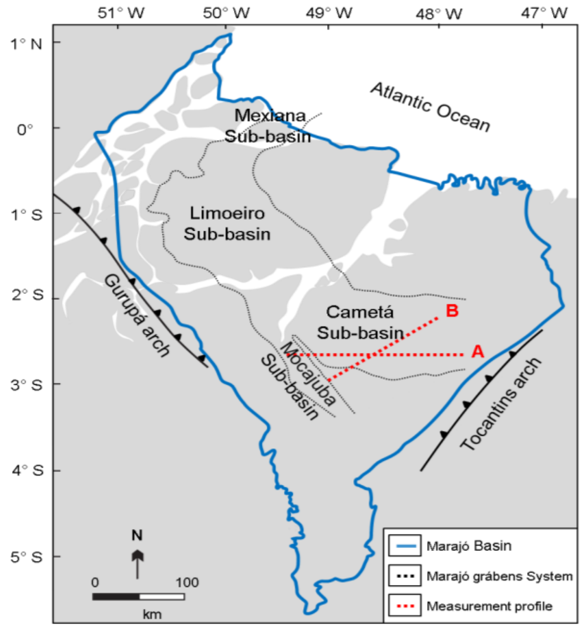

Figure 3 - Location map of profiles A and B used in the inversion process (modified by Costa et al., 2002).

The treatment used in the gravity data of the Marajó Basin follows what was exposed by dos Santos and Júnior (2017). In this approach, we initially perform an interpolation using the kriging method with semivariogram modeling. The regional-residual separation was carried out in two phases, the first of which consisted of: *i*) removing the influence of Moho's discontinuity by applying the upward continuation technique based on the knowledge of Moho's average depth information of the region equal to 32 km obtained from the GEMMA project (GOCE Exploitation for Moho Modeling and Application), shown in figure 4 and *ii*) subtract the continuous and interpolated anomalies resulting in the residual anomalies map (Figure 05).

In the second phase, we separate the influences from internal sources of the sedimentary package from the anomaly due only to the basement, performed through the spectral analysis method (Spector and Grant, 1970). We only preserve the influence of the interface between the basement and the sedimentary package by applying the low-pass filter, responsible for conserving low-frequency signals. As a result of the processing, we compute the residual Bouguer gravity map of the Marajó Basin whose anomalies are produced exclusively by the contrast of density between the sediments and the basement (Figure 06).

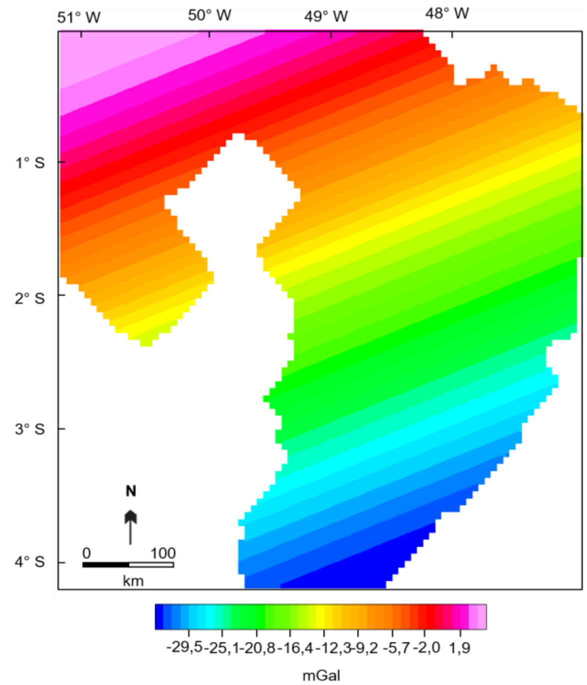

Figure 4 - Map of gravity anomalies in the Marajó Basin for upward continuation of 32 km.

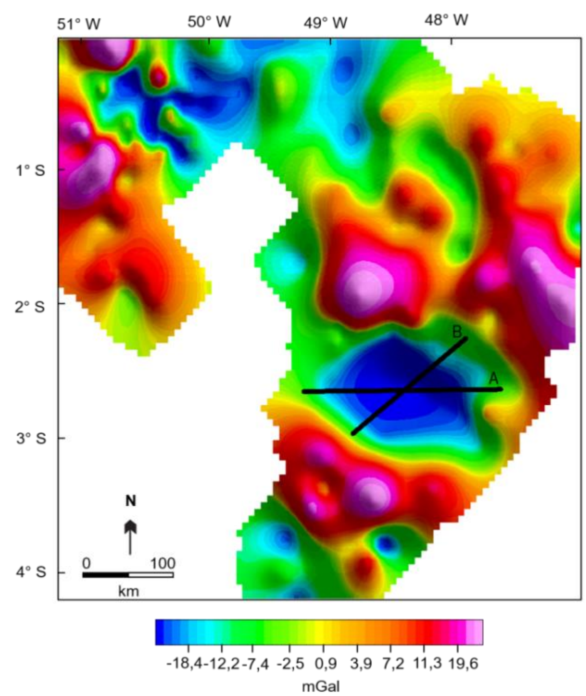

Figure 5 - Map of residual anomalies in the Marajó Basin without the influence of Moho, showing the locations of profiles A and B.

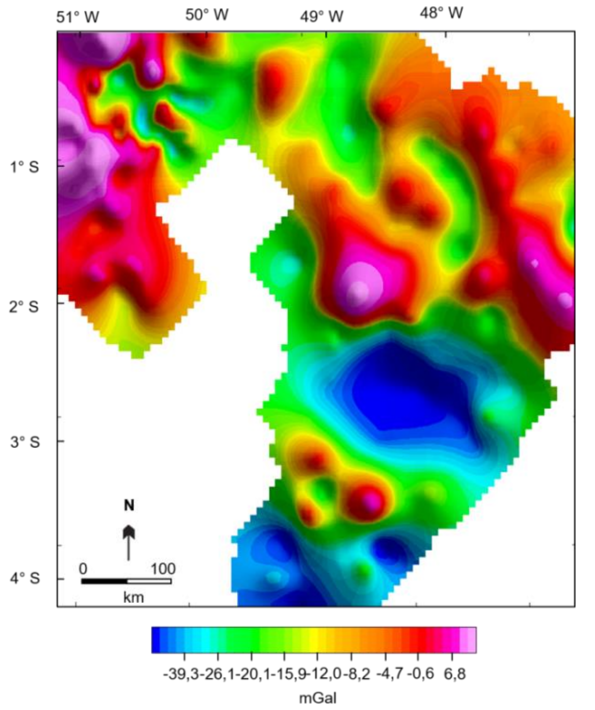

Figure 6 - Gravity map assumed to be produced by the basement of the Marajó Basin.

To apply the proposed inversion method we use profiles A and B extracted from the Bouguer anomaly map shown in Figure 6. The profiles were selected from the analysis of the values of negative anomalies that represent a contrast of lower density than the crystalline basement, since the negative inflections can be related to structures and geological faults.

In order to produce the estimates of the depths of the basement relief, we defined the parameters that control the hyperbolic decay $\Delta\rho_0$ and β through the systematic search method similar to that presented by Silva et al. (2006), associated with information on the maximum basement depth of 11 km (Lima, 1987). As a result, we obtained the estimates $\Delta\rho_0$ = - 0.35 g / cm^3 and β = 4 km.

Profile A is 193 km long, distributed in 93 observations (Figure 7a). We applied the inversion method to the profile using the Total Variation function with μ = 0.3. The interpretative model used contains 93 rectangular 2D prisms juxtaposed along the horizontal extension of profile A, with constant widths of 1 km. The solution has a basement relief that shows the presence of a depression with well-defined discontinuities produced by faults (Figure 7b). This gravity interpretation produced an acceptable RMS value of the data misfit equals to 0.1 mGal. The maximum estimated depth of the basement is approximately 10 km.

Figure 8a shows the profile B containing 57 observations that were inverted using the VT with μ = 0.5 and produced the basement relief estimate (figure 8b) at a maximum depth of 11 km. The solution has a relief with local discontinuities produced by faults. This gravity interpretation produced an acceptable RMS value of the misfit data equals to 0.1 mGal, using 57 overlapping 2D rectangular prisms along the horizontal extension of profile B.

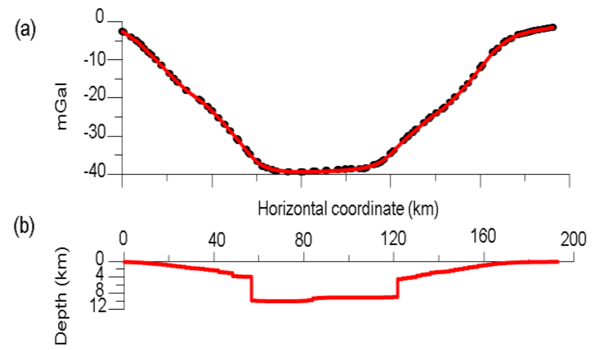

Figure 7 - Marajó Basin. (a) Gravity anomaly (black dots) and adjustment of the anomaly (red line) along profile A (see the location in Figure 5). (b) Estimated relief (red line) obtained with the proposed method with Δρ0 = - 0.35 g / cm3, β = 4 km and μ = 0.3.

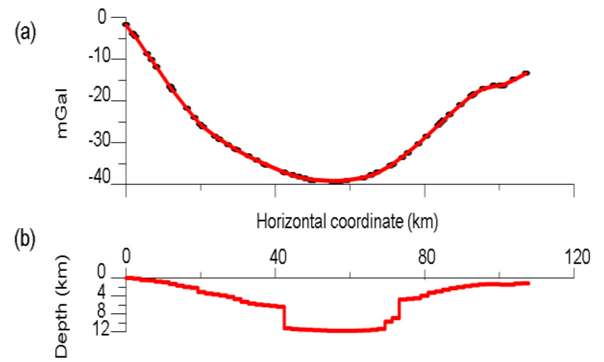

Figure 8 - Marajó Basin. (a) Gravity anomaly (black dots) and adjustment of the anomaly (red line) along profile B (see the location in Figure 5). (b) Estimated relief (red line) obtained with the proposed method with Δρ0 = - 0.35 g / cm3, β = 4 km and μ = 0.5.

According to Villegas (1994) in the Cametá Sub-Basin, there are normally dominant synthetic faults, resulting from the lowering of the ceilings of the master faults and / or greater movement of the prisms along the main fault detachment plans. The inversion results (Figures 4.6 and 4.7) show a basement relief with the presence of discontinuities caused by faults that, according to Costa et al. (2002) are closely related to the complex patterns of shear zones in the Araguaia and Tumucumaque belts. Additionally, the signatures of anomalies of positive and negative severity are related to the density contrast of these belts, which are formed by a wide variety of lithostratigraphic units, which correspond to high and low density rocks, in addition to sliding ductile shear zones.

Conclusion

Studies on the Marajó Basin should be intensified through the acquisition, processing and interpretation of geology, geophysics and geochemistry data. As it is a region with an extensive negative gravity anomaly, two representative profiles were drawn to estimate the outline of the basement relief. In order to contribute with information about the relief of the basement, we selected the gravity profiles A and B of the Bouguer anomaly map from the analysis of the negative anomaly values that represent a lower density contrast than the crystalline basement.

The estimated relief obtained via the 2D inversion method is consistent with the geology, showing the presence of well-defined discontinuities caused by the presence of faults. According to the geological studies of the region, the Araguaia and Tumucumaque belts influence the geometry of the faults found in this basin. Additionally, these structures are related to the negative values of the gravity anomaly. The results obtained produced an acceptable RMS value of the data misfit equals to 0.1 mGal. We estimate that the maximum basement depth is around 11 km, consistent with the estimated depth found in the literature.

Acknowledgments

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